ABSTRACT
Generating Digital Elevation Models (DEMs) out of interferometric image pairs is one of the well-known SAR applications which are used all over the world. The aim of our project is to find new/extended methods for InSAR processing to improve the quality of the final DEM result. We investigate in the fields of image registration, baseline estimation/interpolation and phase unwrapping. In each discipline there is a lot of potential for improving the individual results. The paper gives a summary of the methods we have applied and shows some results using TerraSAR-X test sites.

Index Terms— SAR interferometry, image registration, information measures, orbit interpolation, phase unwrapping, Kalman filter

1. INTRODUCTION
Additionally to the classic correlation based methods, we will employ, compare and combine theoretical methods, based on information theory, such as mutual information, alignability in conjunction with automated bin size determination to achieve optimally results in image registration. Considering the co-registration of single look complex SAR data, where correlation approaches are standard, the exploitation of similarity measures and transformation, based on information theory, is rather novel. Comparing these novel approaches with standard references with respect to quality and quantitative measures, will broaden the scientific understanding, and, furthermore, provide new insights concerning co-registration techniques for interferometric applications. By achieving a more accurate registration of interferometric pairs the phase error will be reduced because the coherence will be improved.

Baseline estimation and interpolation for flat earth removal and height generation: First we want to improve the given position and velocity data by a Kalman filtering and smoothing approach, which uses acceleration information at the actual satellite’s position for correcting and interpolating the orbit data. Besides the available orbit state vector measurements, the Kalman filter incorporates realistic models for gravitation and air drag, thus enabling very accurate orbit estimation, propagation and interpolation. After applying this algorithm to the first and to the repeated orbit, the geometric baseline vector can be calculated as the difference vector of two refined orbit positions; it can be further transformed into the interferometric baseline and then be further improved by using co-registration vectors, determined during the co-registration process.

Phase Unwrapping: Generally a Kalman filter is a powerful tool to obtain accurate model based estimates out of different sources of information. All the given information is fused in an efficient way and also the noise is cancelled optimally. Because of this a Kalman filter based data fusion approach to unwrap and simultaneously filter the phases of interferometric SAR images is developed. The data fusion concept exploits phase information, extracted from the complex interferogram rather than from the phase image and fuses that information with the coherence matrix and the phase slope information extracted from the power spectral density of the interferogram. It is not necessary to generate a phase error based on complicated statistics neither doing phase noise reduction; phase unwrapping takes place simultaneously with removing the phase errors.

2. THE METHOD OF IMAGE REGISTRATION
Image registration is known as an important part of generating Digital Elevation Models (DEM) with Interferometric SAR (InSAR) processing and is one of the critical preprocessing steps in remote sensing. It is used in the formation of 3-D models based on 2-D images taken at different points of view as well as for mosaicking applications.

The paper gives an overview of the information measures which can be used for automatic generation of reference points and finding the correspondences in the second image. The approach is validated with simulated and real images generated from the TerraSAR-X satellite.

The correlation coefficient is one of the popular similarity measures used for intra-modality image registration. It measures the similarity by calculating global statistics such as mean and variance. Another widely used similarity measure is the estimated mutual information which does not assume any functional relationship between images to be registered. It measures the redundancy between two images by looking at their intensity distributions. Mutual information represents relative entropy between two sets. In multimodal image registration mutual information is a standard reference. We show that in the registration of images of the same modality, mutual information can be more robust and reliable than the correlation coefficient.
To reduce computational cost, we use automatic subimage selection as a reduction in search data strategy. We propose a measure, called alignability, which shows the ability of a subimage to provide reliable registration. This feature can be more reliable than other subimage selection methods such as using gradient, entropy and variance, e.g. In a first step we will search for appropriate reference points with high entropy. We compare traditional methods (calculating the effective spectral bandwidth, Shannon’s entropy ...) with other (novel) information measures. Later we want to detect these reference points in the second image applying different methods and strategies. For quantizing the errors which occur we generated a data set of image pairs where the differences (translation, rotation, skewness, noise...) between master and slave image are exactly known. After having evaluated the quality of different information measures for image registration, we will also present registration results of real data sets. Using repeat pass images pairs from the recent German TerraSAR-X satellite we have the possibility to register images of high quality and high resolution (up to 1m).

The registration process consists more or less of these four individual steps:

1) Automatical generation of reference points. This involves the extraction of features to be used for the matching process. Some distinctive features include edges, contours and lines of intersection.

2) Feature matching. In this section the correspondence between the features detected in the input image and features detected in the reference image. Similarity metrics such as mutual information and feature descriptors are used for this purpose.

3) Transformation model estimation. The mapping functions between the input and reference images are estimated by matching the corresponding features.

4) Image resampling and transformation. The input image is transformed according to the estimated mapping function.

In this paper the first two points are considered. These steps could be repeated for applying course and fine registration. The reference points should be selected regarding their value of information and optimal geometrical distribution aspects.

2.1. Information measures for the generation of reference points

2.1.1. Shannon Entropy

The Shannon Entropy is a common information measure of the uncertainty associated with a random variable.

\[ I_{\text{Shannon}} = \sum_{k=1}^{\infty} p_k \cdot \ln \left( \frac{1}{p_k} \right) = -\sum_{k=1}^{\infty} p_k \cdot \ln p_k \]  

(1)

The entropy of a random variable is defined in terms of its probability distribution and can be shown to be a good measure of randomness or uncertainty. A subimage with a sharp probability density distribution correspond to low entropy, whereas a dispersed distribution yields a high entropy value.

2.1.2. Alignability

Alignability is the ability of an image to provide reliable image registration results by showing the correct transformations. Alignability \( A_y \) is computed by mutual information values of a subimage against the transformed version of itself. A set of mutual information (2.2.2) values, \( M I \ S_i \) are obtained by comparing the subimage \( S_i(\theta) \) by a rotated version of itself, \( S_i(0) \),

\[ MI_{S_i} = MI(S_i(0), S_i(\theta)) \]  

(2)

where \( \theta \) is the angle of rotation. In our work, \( \theta \) is varied between -10° to 10° with an increment of 1°. The scaled difference of highest and the second highest MI values is defined as alignability. This can be expressed as

\[ A_y = MI_1 - MI_2 \]  

(3)

2.1.3. Spectral Bandwidth

For optimizing the registration results using the correlation coefficient (2.2.1) we have implemented a measure for the effective spectral bandwidth. From the signal theory we know that the larger the bandwidth, the sharper the distribution of the correlation coefficient. For this reason we have to find the cutout of the image which has the biggest value for the spectral bandwidth. This gives a higher chance for detecting this cutout in the second image.

Starting from the energy density spectrum (EDS)

\[ \phi_v(f) = |S(f)| \]  

(4)

which has to be normalized,

\[ \tilde{\phi}_v(f) = \frac{\phi_v(f)}{\int_{-\infty}^{\infty} \tilde{\phi}_v(f)df} \]  

(5)

the total power could be calculated.

\[ P_f = \int_{-\infty}^{\infty} f^2 \cdot \tilde{\phi}_v(f)df \]  

(6)

The normalized EDS is positive definite, has a per unit area and features in this way the main properties of a probability density function (PDF). From this the mean

\[ \mu_f = \int_{-\infty}^{\infty} f \cdot \tilde{\phi}_v(f)df \]  

(7)

, the covariance

\[ \sigma_f^2 = P_f - \mu_f^2 = \int_{-\infty}^{\infty} (f - \mu_f)^2 \cdot \tilde{\phi}_v(f)df \]  

(8)

and the spectral bandwidth

\[ \sigma_f = \sqrt{P_f - \mu_f^2} = \sqrt{\int_{-\infty}^{\infty} (f - \mu_f)^2 \cdot \tilde{\phi}_v(f)df} \]  

(9)

can be calculated.
2.2. Similarity metrics

2.2.1. Correlation Coefficient.

The correlation coefficient (CC) is one of the most widely used similarity metrics. It is a second order metric defined by

\[ \rho(A, B) = \frac{E[(A - \mu_A)(B - \mu_B)]}{\sigma_A \sigma_B} \]  

(10)

where A and B are portions of the master and the slave image.

2.2.2. Mutual Information

Mutual information (MI) has emerged in recent years as a popular similarity metric in the registration of images (especially in the field of medical applications). If we term the entropy of random variable A

\[ H(A) = \sum_{a \in A} p_a(a) \log \frac{1}{p_a(a)} \]  

(11)

and the joint entropy of A,B

\[ H(A, B) = - \sum_{a \in A, b \in B} p_{ab}(a,b) \log p_{ab}(a,b) \]  

(12)

the mutual information between two random variables A and B is given by

\[ MI = \sum_{a \in A, b \in B} p_{ab}(a,b) \log \frac{p_{ab}(a,b)}{p_a(a)p_b(b)} \]  

(13)

In terms of entropy and joint entropy we can write

\[ MI = -H(A, B) + H(A) + H(B) \]  

(14)

Often the normalized version of mutual information is used

\[ NMI = \frac{MI(A, B)}{H(A, B)} = \frac{MI(A, B)}{H(A, B)} + 1 \]  

(15)

which gives only values between 0 and 1 and simplifies the interpretation the obtained values of the mutual information. The maximization of mutual information criterion postulates that mutual information is maximized if images are correctly registered.

2.2.3. \( \chi^2 \) divergence

\( \chi^2 \) divergence (distance to independence) exclusively uses the estimation of the joint probability density function and does not use the radiometries of the pixels. Distance to independence is a normalized version of the \( \chi^2 \) test [3]:

\[ \chi^2(A, B) = \sum_{i,j} \left( \frac{(p_{ij} - p_a p_b)^2}{p_a p_b} \right) \]  

(16)

It measures the degree of the statistical dependence between two images.

2.3. The influence of the bin size

The bin size of the histogram used in the estimation of mutual information is a critical issue. The reliability and robustness is enhanced if the right bin size is chosen.

![Figure 1: Diagram of incorrectly registered sub images (Mutual Information) for different simulated scenes and bin sizes](image)

![Figure 2: Normalized value of MI using different bin sizes](image)

Choosing more bins allows for a more detailed representation of probability densities. However, this detail may be nothing more than noise, caused by small sample size in each bin. In the past, investigators have used fixed bin size which was determined empirically. We chose to formalize the approach by using a variable number of bins, calculated by Sturges' formula, Scott's or Freedman-Diaconis' choice.

\[ h_{st} = 1 + \log_2 n = 1 + 3.3 \log_{10} n \]  

(17)

While the Sturges’ formula only depends on the size of the dataset, the rules of Scott or Freedman/Diaconis depends also on the characteristics of the histogram:

\[ h_{scott} = 3.49 \cdot \sigma \cdot n^{-1/3} \]  

(18)

For example Scott’s rule uses the standard deviation of the histogram; Freedman-Diaconis’ choice is based on the interquartile range.

\[ h_{FD} = 3.49 \cdot (Q_3 - Q_1) \cdot n^{-1/3} \]  

(19)

In figure 3 the influence of the bin size of the histogram is shown. The four bars for the different bin sizes stand for using translated, noisy, rotated and rotated + scaled image pairs.

![Figure 3: Mismatching results for different data sets and bin size](image)
2.4. Simulation Results

To number the quality of our registration approach we have generated simulated InSAR image pairs out of real TerraSAR-X data.

The advantage of a simulation is that we can exactly measure how accurate our method is. We create image pairs mutated by translation, rotation, scaling or noise. First we compare the different information measures for automatic reference point generation listed up in 2.1. Shannon Entropy is the fastest method, but regarding the correct matching of the reference cutout in the second image the spectral bandwidth method and alignability give the best results.

As similarity measures we used the correlation coefficient (CC), Kolmogorov distance (KD), mutual information (MI) and chi² divergence:

<table>
<thead>
<tr>
<th>Measure</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolmogorov Distance</td>
<td>½</td>
</tr>
<tr>
<td>Mutual Information</td>
<td>x log x</td>
</tr>
<tr>
<td>χ² Divergence</td>
<td>½ (x-1)²</td>
</tr>
</tbody>
</table>

Table 1: Comparison of the χ² Divergence Family

For comparing the similarity metrics figure 5 is used. In this experiment the image pairs are always rotated (2°), then scaling, noise and a higher rotation angle (6°) are added.

Figure 6 shows the computational cost of the different methods.

Obviously the traditional correlation coefficient is the most cost expensive way the compare the cutouts of the images. The discrepancy arises the larger these cutouts are.

Figure 7-9: Image shift experiment with translated (2), noisy (3) and rotated+scaled+noisy (4) data

If there is no big difference between the two chosen cutouts chi² divergence is a good choice; regarding noise, scaling and rotation it often leads to misregistration.
dissimilarity between the two images. All methods seem to be robust to noise; chi² divergence fails in the case of rotation and scaling.

3. ORBIT INTERPOLATION

The first step of our method of orbit interpolation is to generate an orbit model based on a realistic gravitation model. By calculating the gradient of the gravitation field we obtain the acceleration that influences the satellite. The integration of this acceleration is done by a Kalman filter that contains a state space model where position, velocity, acceleration and its derivatives up to the 3rd order are included. The acceleration, which is calculated by evaluating the gravitation model at the predicted position of the satellite, serves as observation. Thus a predictor – corrector structure is performed. Simulated noisy GPS position measurements are included by simply adding them to the observation vector.

For the calculation we use two different coordinate frames:

3.1. The used Frames

The earth centered inertial frame is defined as follows:

\[ \tilde{r}_{IN} = x_{IN} \cdot \tilde{e}_{IN} + y_{IN} \cdot \tilde{e}_{YN} + z_{IN} \cdot \tilde{e}_{ZN} \] (20)

where the mass center of earth is the point of origin, the x-axis points towards the vernal equinox, the z-axis points towards the north pole and the y-axis completes the right handed Cartesian frame.

The vectors in this frame can also be converted into polar coordinates \( \varphi, \theta, r \), where \( \varphi \) is the angle in the x-y-plane measured from the x-axis towards the y-axis (range=[0..2\pi]), \( \theta \) is the angle between the vector and the z-axis measured from the z-axis towards the vector (range=[0..\pi]) and \( r \) is the norm of the vector (range=[0..\infty]). For the calculations in this paper \( \theta \) is converted into a range of \([-\pi/2..\pi/2]\] to match the geocentric latitude in the earth centered earth fixed frame.

The earth centered earth fixed frame is defined by:

\[ \tilde{r}_{EF} = x_{EF} \cdot \tilde{e}_{x_{EF}} + y_{EF} \cdot \tilde{e}_{y_{EF}} + z_{EF} \cdot \tilde{e}_{z_{EF}} \] (21)

where again the mass center of earth is the point of origin. The x-axis points towards the intersection of the zero meridian and the equator. The z-axis is the same as in the earth centered inertial frame and the y-axis completes the right handed Cartesian frame.

Compared to the inertial frame the earth fixed frame rotates around the z-axis with an angular rate of \( \omega=7.292115 \) rad/s.

In some cases the Cartesian coordinates are expressed in geocentric longitude \( \lambda \) (range=[0..2\pi]), geocentric latitude \( \delta \) (range=[-\pi/2..\pi/2], \( \delta=\pi/2 \) at the north pole) and distance to the point of origin \( r \) (range=[0..\infty]).

3.2. The Gravitation Model

A good description of the earth’s gravitation can be achieved by a harmonic analysis of the potential [6].

The equation is:

\[ U = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{\mu \cdot a_n^2}{r^{n+1}} \] (22)

\[ \sum_{m=0}^{n} \bar{P}_{nm}(\sin(\delta)) \cdot (c_{nm} \cdot \cos(m\lambda) + s_{nm} \cdot \sin(m\lambda)) \]

where:

- \( n \) - grade of the potential model
- \( m \) - order of the potential model
- \( r \) - distance to the center of the Earth
- \( \delta \) - geocentric latitude
- \( \lambda \) - geographic length
- \( \bar{P}_{nm} \) - fully normalized Legendre functions
- \( C_{nm}; S_{nm} \) - harmonic coefficients of the Earth’s potential
- \( a_n \) - semi-major axes of the Earth

The geocoeficients are taken from the CHAMP-only Earth Gravity Field Model EIGEN-2.

Figure 10: Gravitation model of the Earth

The fully normalized Legendre functions are calculated using a recursive algorithm [6]:

\[ \bar{P}_{n,m} = \eta \sin(\delta) \cdot \bar{P}_{n-1,m} - \sigma \cdot \bar{P}_{n-2,m} \quad ; n > m + 1 \]

\[ \bar{P}_{n,n-1} = \tau \sin(\delta) \cdot \bar{P}_{n-1,n-1} \quad ; n > 1 \]

\[ \bar{P}_{n,n} = \nu \cos(\delta) \cdot \bar{P}_{n-1,n-1} \quad ; n > 1 \] (23)

with the start values

\[ \bar{P}_{0,0} = 1, \quad \bar{P}_{1,0} = \sqrt{3} \sin(\delta), \quad \bar{P}_{1,1} = \sqrt{3} \cos(\delta) \]

and

\[ \eta = \frac{(2n+1) \cdot (2n-1)}{(n+m) \cdot (n-m)} \]

\[ \tau = \sqrt{2n+1} \]

\[ \sigma = \sqrt{(2n+1) \cdot (n+m+1) \cdot (n-m+1)} \]

\[ \nu = \sqrt{\frac{2n+1}{2n}} \]

3.2.1. Calculating the acceleration

Because the acceleration depends on the position related to the WGS84 ellipsoid the calculation can not be done in the inertial coordinate frame. To evaluate the gravitation model at the right position the vector that is expressed by inertial coordinates must be transformed into earth centered earth fixed coordinates. This is done by rotation...
around the common z-axis of both the inertial and the earth fixed frame. The rotation angle is
\[ \Theta = \omega t \]
where \( \Theta \) is the date dependent angle between the inertial frame and the earth fixed frame at the starting time of the algorithm. This angle is calculated using the MJD (Modified Julian Date).

To go on with the calculation the so obtained acceleration vector must be transformed back into the inertial frame by
\[
\tilde{a}_I = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} -\sin(\phi) - \sin(\delta) \cos(\phi) \\ \cos(\phi) - \sin(\delta) \sin(\phi) \\ 0 \end{bmatrix} \begin{bmatrix} a_y \\ a_x \\ a_r \end{bmatrix}
\]

### 3.3. Integration

A great advantage of using a Kalman filter is the fact that measurements can be easily included by adding them to the observation. In the present case the state transition matrix corresponds to a Taylor series of the equation of motion. For the one-dimensional case the equation is:
\[
x(k+1) = \begin{bmatrix} s(k+1) \\ v(k+1) \\ a(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & \frac{T}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s(k) \\ v(k) \\ a(k) \end{bmatrix} + \begin{bmatrix} \frac{T^5}{5!} \\ \frac{T^4}{4!} \\ \frac{T^3}{3!} \end{bmatrix}
\]

where \( s \) is the position, \( v \) is the velocity, \( a \) is the acceleration and \( T \) is a fixed discrete time interval.

The observation matrix is adapted in case of arriving position measurements by setting the elements which correspond to the observed position in the state vector to 1. Otherwise all elements are 0 except those that correspond to the observed acceleration.

#### 3.3.1. Measurement Noise

To obtain usable results it is necessary to model the noise of the measurements. This is done by adding a diagonal matrix which contains the squares of the standard deviation (\( \sigma^2 \)) when calculating the Kalman gain. The elements corresponding to the position are set to \( \sigma^2 \) of the simulated position measurements.

The elements corresponding to the acceleration taken from the gravitation model depend on the weight that shall be given either to the orbit model or to the measurements. Basically when using a Kalman filter the noise is assumed to be uncorrelated, Gaussian distributed white noise by approximation.

Another important feature is the driving noise, which indicates the quality of the model on which the Kalman filter is based. It is added at each prediction step and can also be used to give more or less weight to the underlying model.

### 4. PHASE UNWRAPPING

Nearly all known phase unwrapping techniques try to unwrap the mapped phases by a sequence of differentiating, taking the principal value of the discrete derivative and integrating it again. A serious drawback of any differentiation of functions which are modulo mapped and noise contaminated is a bias resulting from the discrete derivative of noisy modulo-2Pi mapped phases.

As a consequence of this bias, phase slopes are always underestimated. The resulting bias depends on the phase slope itself, as well as on the coherence. Our phase unwrapping algorithm is based on an Extended Kalman filter. The Kalman filter exploits a so called "Basic – Slope Model" enabling the filter to incorporate additional local slope information obtained from the sample frequency spectrum of the interferogram by a local slope estimator. The local slope information is then optimally fused with the information directly obtained from real and imaginary part of the interferogram. For this reason it is not necessary to generate a phase error based on complicated statistics, phase unwrapping takes place simultaneously with removing the phase errors. Whereas some techniques try to reduce the phase noise by filtering before unwrapping the phase, the Kalman filtering approach simultaneously unwraps the phases and eliminates the phase noise, so that no pre-filtering is necessary.

Based on our experience in this area [8], we have done some refinements for improving the unwrapping results. There some methods for reducing the computational load of our algorithm are given as well as a new way to use a weighted 2D Kalman filter. Therefore the filter can yield excellent results of the unwrapped phase even in regions with steep and rough topography.

#### 4.1. The Interferogram

The phases of an InSAR image are all mapped into the same 'baseband' interval (e.g. \(-\pi, \pi\)), while any absolute phase offset (an integer multiple of \(2\pi\)) is lost. Furthermore they are subject to phase noise caused by the superimposed amplitude noise in the real and imaginary parts of the InSAR image.

For the complex SAR interferogram we have in polar notation at point \((n,m)\):
\[
z(n,m) = a(n,m) \cdot \exp\{j\phi(n,m)\}
\]
with \(a(n,m)\) being the observed interferometric amplitude and being the interferometric phase modulo mapped, where the modulo mapping is generally expressed by:
\[
\tilde{\alpha} = [\alpha]_{2\pi} = \alpha \pm n \cdot 2\pi \in (-\pi, \pi] \quad \text{and} \quad |\alpha| \leq \pi
\]
Simplifying the notation to one-dimensional position dependence we write:
\[ \tilde{\phi}(k) = \left[ \phi(k) + e_{\phi}(k) \right] \bmod \! 2\pi \]
\[ = \left[ \phi(k) + e_{\phi}(k) \right] \bmod \! 2\pi \]
\[ = \left[ \phi(k) + e_{\phi}(k) \right] \bmod \! 2\pi \]
where is the true unambiguous phase at time or point \( k \), is the true phase error is the mapped phase error at point \( k \), the stochastic parameters, such as probability density.

4.1. The Principle of the algorithm

Based on the specific model of the interferometric phase an extended Kalman filter is implemented, which additionally fuses phase slope estimation, calculated from the interferogram’s power spectral density.

4.2. The Slope Estimator

The interferometric phase at time \( t \) is decomposed into the sum of three terms:
\[ \phi(t) = \phi(t_0) + \int \phi'(t) \, dt = \phi(t_0) + 2\pi \cdot \int (f_0 + \tilde{f}(\tau)) \, d\tau \]
\[ = \phi(t_0) + 2\pi \cdot f_0 \cdot (t - t_0) + 2\pi \cdot \int \tilde{f}(\tau) \, d\tau \]
(28)

Substituting this decomposition into (1) with normalized amplitude and using complex notation we have:
\[ z(t) = \exp \left\{ j \left[ \phi(t_0) + 2\pi f_0 (t - t_0) + 2\pi \int \tilde{f}(\tau) \, d\tau \right] \right\} + n(t) \]
\[ = \exp \left\{ j 2\pi f_0 (t - t_0) \right\} \cdot \exp \left\{ j 2\pi \tilde{f}(\tau) \right\} + n(t) \]
(29)

The mean fringe frequency \( f_0 \), corresponding to the mean phase slope with respect to a given observation window can be observed as a spectral shift in the interferogram’s power spectral density. Hence we might use any local frequency estimator and apply it to the complex interferogram, to estimate the spectral shift from the complex correlation kernel of the interferogram. Likewise we could also apply any other local frequency estimation, based on the fact that the instantaneous frequency can be directly calculated from the complex data by:
\[ f(t) = \frac{d}{dt} \phi(t) = \frac{1}{2\pi} \frac{dz(t)}{dt} \quad \text{where: } z(t) = \frac{dz(t)}{dt} \]
(30)

Rather than that, we will estimate the spectral shift in the frequency domain from the interferogram’s power spectral density, obtained in a locally shifted window. The technique is rather conventional, calculating the power spectral density in a local window and applying some subpixel resolution technique to identify the maximum with subpixel resolution. Good results for finding the position of maximum of the interferogram’s power spectral density with subpixel accuracy gives a method where we fit the original curve with a quadratic curve. After minimizing the quadratic error of original and fitted curve we can determine the needed frequency which corresponds to the phase slope. A drawback of the slope estimation technique is the high computational cost. For a better performance of the complete algorithm we found ways to save loops or make individual procedures faster.

Because most of the information of the new (sliding) window is also contained in the previous window we can subtract the not required information of one row/column in the Fourier domain and add the new row/column information. This does not affect the quality of the result, but increases the processing speed.

4.3. The Kalman filter

For solving the non-linear phase unwrapping problem a 2D Kalman filter was implemented, which can be derived from the stand equations of a 1D extended Kalman filter. The algorithm has a recursive predictor-corrector structure; the high-ranking “…” and “…”-signs describe the point in time. While “…” means short before the income of the measurements, “…”-signs describe the point in time when the new measurements are available for the actual time index \( k \).

\[ \hat{x}_{k+1} = A \cdot \hat{x}_k + B \hat{u}_k \]
\[ P^{-}(k+1) = A \cdot P^{-}(k) \cdot A^T + Q(k) \]
(31)
\[ K(k + 1) = P^-(k + 1) \cdot C_p^T(k + 1) \cdot C_p(k + 1) \cdot P^-(k + 1) + R(k + 1) \]

\[ L_{k+1} = \frac{y_{k+1} - h(x_{k+1})}{\sqrt{P_{k+1}^T} + K(k + 1) \cdot L_{k+1}} \]

\[ P'(k + 1) = P^-(k + 1) - K(k + 1) \cdot C_p^T(k + 1) \cdot P^-(k + 1) \]

A is the state transition matrix, describing the dynamical changes of the variable, which is to be estimated with respect to time or space, u is any deterministic or known influence changing the state \( x(k) \) from one pixel to the next \( x(k+1) \). R(k+1) is the measurement noise covariance and Q(k) the driving noise covariance describing the uncertainty of the state transition.

\[ P \] is the covariance matrix which expresses a measure for the discrepancy of the predicted/updated estimate to actual state. The linearized observation matrix is given by:

\[ C_p(k + 1) = \frac{d}{d\tilde{x}} h\tilde{x} \bigg|_{\tilde{x}_{k+1}} \]  

(32)

Due to the nonlinear observation mapping, the Kalman filter will automatically unwrap the interferometric phases. In doing so, the Kalman filter fuses the information gained from the complex interferogram with the slope information extracted from the interferometric power spectrum.

### 3. WEIGHTED 2D KALMAN FILTERING

In the case of phase unwrapping we have, a scalar state and a vectorial observation, containing the complex interferogram pixel.

Due to the nonlinear observation mapping, the Kalman filter will automatically unwrap the interferometric phases. The Kalman filter fuses the information gained from the complex interferogram with the slope information extracted from the interferometric power spectrum.

For the two-dimensional unwrapping the Kalman filter algorithm works along the sketched integration paths. Any prediction estimate is calculated, depending on two neighbors (figure 12), so that only the prediction equations in the Kalman filter algorithm have to be modified. The same applies for the error covariance matrix, which is a weighted sum of the two involved neighbored covariance matrices.

\[ \hat{x}(r-1,a) \]
\[ u(r-1,a) \]

\[ \hat{x}_1(r,a), P'_1(r,a) \]
\[ \hat{x}_2(r,a), P'_2(r,a) \]

\[ \hat{x}^+(r,a-1) \]
\[ u(r,a-1) \]

1D Kalman filter

1D Kalman filter

\[ \hat{x}^+(r,a) \]
\[ \hat{x}_1(r,a), P'_1(r,a) \]
\[ \hat{x}_2(r,a), P'_2(r,a) \]

In our approach the control vector \( u(k) \) corresponds to the estimated slopes in range and azimuth direction and includes also adaptive calculated variance (\( w(k) \)) to describe the uncertainty of the estimates.

\[ u_r(r,a) = \Delta \hat{\phi}_r(r,a) + w(r,a) \]
\[ u_a(r,a) = \Delta \hat{\phi}_a(r,a) + w(r,a) \]  

(33)

For improving the algorithm presented in [6] we added some weighting factors to obtain the best possible estimate of the unambiguous phase.

\[ z(r,a) = f(x(r,a)) \]

(34)

We obtain the linearized observation matrix by the vectorial derivation of the given nonlinear observation \( h(x) \).

After comparing the actual observation with the predicted state, we can compute the best possible updated state estimate for the phase for each individual pixel (figure 13).

### 4. OPERATION IN AREAS OF LOW COHERENCE

The result of the unwrapped product will be dramatically reduced in quality if the coherence (e.g. caused by radar show, layover, vegetation …) is regionally low. Because of error propagation the filter can never return back to the correct phase. For getting rid of this problem we have implemented several methods.

The coherence
\[ \gamma(r,a) = \frac{E\left\{ \mathbf{z}_2(r,a)^* \mathbf{z}_1(r,a) \right\}}{\sqrt{E\left\{ \mathbf{z}_2(r,a)^2 \right\} E\left\{ \mathbf{z}_1(r,a)^2 \right\}}} \]  

is also an important parameter to quantize the measurement noise (see figure 13) or to serve as an additional source of information for our state space model.

Because of our analysis of the regarded interferogram we have a lot of information (slopes, variance of the slope, noise prediction) we can use. Together with other information (like coherence) we are able to automatically mask out areas in which our algorithm will probably give wrong phase estimates. According to this only a small part of the interferogram is neglected by the processor while the most areas are processed correctly. If there is some a priori knowledge available (SRTM or other available DEMs) we can take this information and complete our phase slope matrix, which otherwise will be filled with zeros. Driven by this model the Kalman filter can fill the gaps of the masked out areas.

Another solution to avoid phase errors caused by areas of low coherence is to use a Kalman smoother: We can start the 2D filtering process from each edge of the interferogram in four different directions.

The filter estimate of the individual filters is then combined with the prediction estimate of the other filters where the weighting of the individual estimates is inversely proportional to the corresponding error covariances.

\[ \hat{x}_k = P_f(k)^{-1} \left( P_f(k) \hat{x}_f(k) + P_b(k)^{-1} \hat{x}_b(k) \right) \]  

\[ P_f(k) = P_f(k-1) \cdot \left( P_f(k) + P_b(k)^{-1} \right)^{-1} \cdot P_b(k) \]  

Index \( f \) means forward and index \( b \) means backward direction of the filter.

Though the processing burden results almost completely from the phase slopes estimation step, using a smoother will proportional not raise much the computation time.

Our approach works very well for obtaining the unambiguous phase of a complex interferogram, which was shown in [8] by using simulated data. The high computation time results only from extracting the necessary information out of noisy interferogram. For this reason we have extended this approach to make it more robust and make the phase slope estimation faster. A further improvement of reducing the computational cost is reached by parallel computing.

A new multi-baseline approach for the Kalman filter for eliminating the ambiguities caused by layover is part of our research.

5. RESULTS USING TERRASAR-X DATA

Because of the small baseline of real TerraSAR-X InSAR image pairs the registration of these images was successful in all cases/methods. Since the MI method is fast and robust this method is preferred. This algorithm is faster than the correlation coefficient algorithm and more reliable than the others. The less misregistered points are present the easier is the elimination of the wrong vectors.

In some cases the along track baseline might be large in that way there is also a large variance of the shifting vectors in the scene (figure 14); that means there is not a constant or nearly constant offset in the shift. Especially in this case our automatic algorithm leads to perfect matching of the two images.

As an example we chose a test site of Ayers Rock in Australia (a lot of layover). In figure 14 the shifting vectors are shown. The individual arrows show the direction of the slave pixel to be found in the master image.

![Figure 14: The shifting vectors](image)

![Figure 15: The interferometric phase](image)

![Figure 16: The flattened phase](image)
6. ACKNOWLEDGEMENT

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7. REFERENCES


